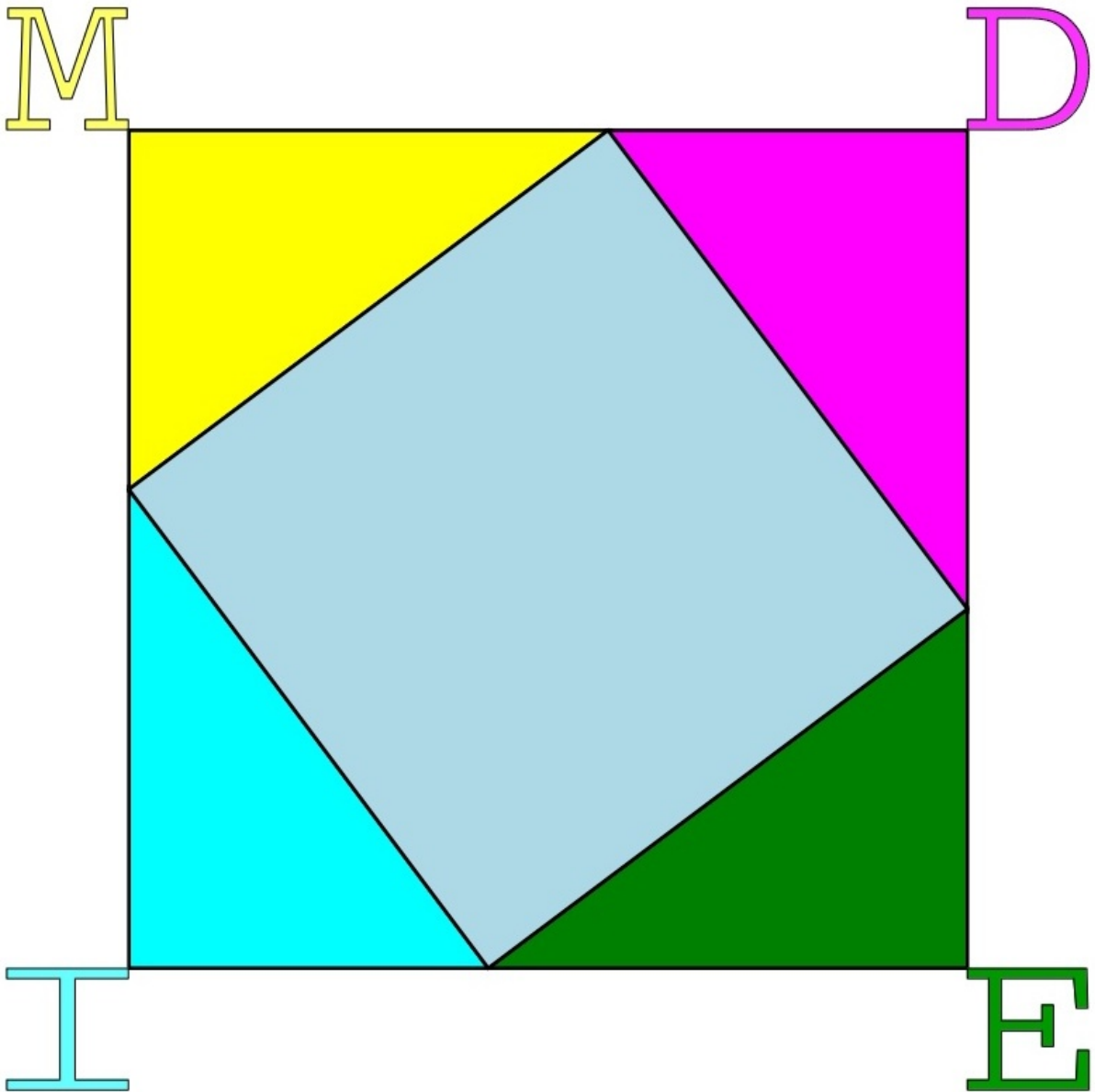# Mathematics Done in English

Douglas Perkins

January 15, 2022

Preface

"Number rules the universe."

– The Pythagoreans

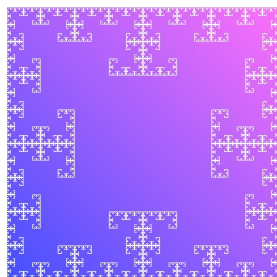*"It matters little who first arrives at an idea, rather what is significant is how far that idea can go."*
– Sophie Germain

"Without mathematics, there's nothing you can do. Everything around you is mathematics."
– Shakuntala Devi

"A great discovery solves a great problem but there is a grain of discovery in the solution of any problem. Your problem may be modest; but if it challenges your curiosity and brings into play your inventive faculties, and if you solve it by your own means, you may experience the tension and enjoy the triumph of discovery."
– Georg Polya, "How To Solve It" (1945)

Table of Contents

Western Arabic numerals

1   2   3   4   5   6   7   8   9   10

Eastern Arabic numerals

١   ٢   ٣   ٤   ٥   ٦   ٧   ٨   ٩   ١٠

Roman numerals

I   II   III   IV   V   VI   VII   VIII   IX   X

Standard Japanese numerals

一   二   三   四   五   六   七   八   九   十

Formal Japanese numerals

壱   弐   参   四   五   六   七   八   九   拾

Mayan numerals

Thai numerals

๑   ๒   ๓   ๔   ๕   ๖   ๗   ๘   ๙   ๑๐

Babylonian numerals

Part I: Numbers

Dice. Picture by CGP Grey.

Chapter 1: Counting

Telephone Interview

- Ask your classmates their telephone numbers.

- How fast can you say your own phone number?

German telephone, 1972.

The Unlucky 21 Game

- Make pairs.

- Start at 1 and count to 21.

- Each player can say 1, 2, 3, or 4 numbers.

- The person who says "21" loses.

- **Question**: The second player can always win. How?

Two Kinds of Numbers

Cardinals	one	two	three	four	five	six	...
Ordinals	first	second	third	fourth	fifth	sixth	...

Fill in the Blanks

1. Mark has _____ brother and no sisters.

2. The science room is on the _____ floor.

3. Five plus ten is _____.

4. Her birthday is on Halloween. That's October _____.

5. There are two outs and runners on _____, _____, and _____.

Number Comprehension

Practice listening to and saying the numbers.

A. 0	D. 1,024	G. 65,536
B. 575	E. 1,112	H. 1,048,576
C. -37	F. 6,536	I. 10,048,576

Card Counting

Take a deck of cards and total the numbers.

- Make a group. Go around clockwise.
- Start with zero.
- Flip over the top card.
- Add it to the total, and say the number.
- Repeat until all the cards are gone.
- Question: What is the total for the whole deck?

Sequences

What numbers come next?

A.	1	2	3	4	5	6	→	____	____	____
B.	2	4	6	8	10	12	→	____	____	____
C.	5	10	15	20	25	30	→	____	____	____
D.	1	3	9	27	81	243	→	____	____	____
E.	2	3	5	7	11	13	→	____	____	____
F.	10	20	60	70	110	120	→	____	____	____
G.	3	1	4	1	5	9	→	____	____	____
H.	31	28	31	30	31	30	→	____	____	____

Sequence Making

Make some new sequences and show the class.

I.	____	____	____	____	____	____	→	____	____	____
J.	____	____	____	____	____	____	→	____	____	____

Minimal Pair Listening

Circle the word you hear.

1. first / fast		6. three / tree	
2. thirteen / thirty		7. some / sun	
3. fourteen / forty		8. third / sad	
4. nineteen / ninety		9. big / bag	
5. tens / tense		10. many / money	

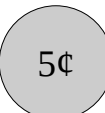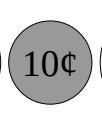# Matching

Pair the digits with the words.
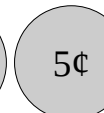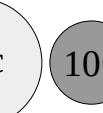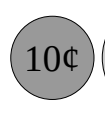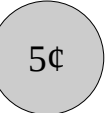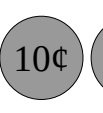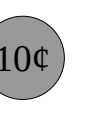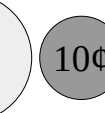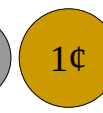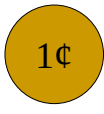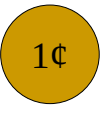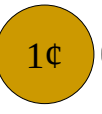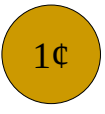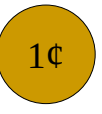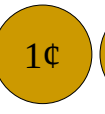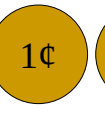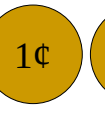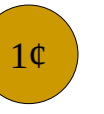
1.	____	eleven	A.	11
2.	____	minus two	B.	110
3.	____	negative five	C.	$^1/_3$
4.	____	one hundred ten	D.	600,000
5.	____	one half	E.	-2
6.	____	one third	F.	0.51
7.	____	six thousand	G.	6,000
8.	____	zero point five one	H.	60,000
9.	____	sixty thousand	I.	0.7
10.	____	point seven	J.	$^{22}/_7$
11.	____	six hundred thousand	K.	-5
12.	____	twenty-two over seven	L.	$^1/_2$

A Coin Game

- This is a two-player game: Player A and Player B.

- Player A goes first, then Player B, then Player A, and so on.

- Each turn, the player takes one coin from the table.

- Players can only choose coins on the left and right sides.

- The player with the most money at the end is the winner.

- Which player do you want to be?

1. 1¢ 5¢ 10¢ 25¢	2. 5¢ 5¢ 25¢ 10¢
3. 10¢ 5¢ 10¢ 10¢	4. 5¢ 25¢ 10¢ 1¢
5. 1¢ 1¢ 1¢ 25¢ 1¢ 1¢ 1¢ 1¢ 1¢ 1¢	

Number Questions

1.	How much is five times itself?	25	55	85
2.	What number times itself equals four hundred?	10	18	20
3.	What is the largest even number less than ninety-nine?	10	98	100
4.	What number plus twice itself equals thirty-six?	8	12	16
5.	What number times anything equals itself?	0	1	π
6.	How much is one half plus one and a half?	2	2.5	3
7.	How many seconds are in two minutes?	120	200	220
8.	How many seconds are in an hour?	2,400	3,600	36,000

Roman Numerals

1	5	10	50	100	500	1000
I	**V**	**X**	**L**	**C**	**D**	**M**
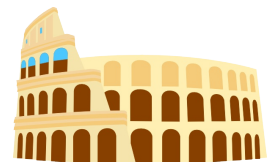
1	2	3	4	5	6	7	8	9	10
I	**II**	**III**	**IV**	**V**	**VI**	**VII**	**VIII**	**IX**	**X**

Roman Numeral Practice

1. How do you write *eleven*? _____
2. How do you write *twelve*? _____
3. How do you write *nine*? _____
4. How do you write *nineteen*? _____
5. How do you write *fourteen*? _____
6. How do you write *fifty-five*? _____
7. How do you write *159*? _____
8. How do you write *1912*? _____
9. How do you write *1982*? _____
10. How do you write *2017*? _____
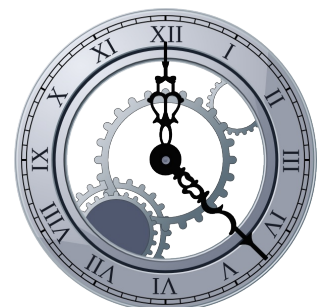
Chapter 2: Chance

heads

tails

Flip a Coin

On average, heads and tails have the same chance of happening. Let's see what happens for us.

Flip #	1	2	3	4	5	6	7	8	9	10
Result	HEADS	HEADS	HEADS	HEADS	HEADS	HEADS	HEADS	HEADS	HEADS	HEADS
	TAILS	TAILS	TAILS	TAILS	TAILS	TAILS	TAILS	TAILS	TAILS	TAILS

Kinds of Dice

We commonly see dice with six sides. There are other kinds, though. Four, six, eight, ten, twelve, and twenty are common. Some board games and card games involve these unusual kinds of dice.

d4   d6   d8   d10   d12   d20

Probability

What is the probability of the following?

A. Roll a d6 and get a 1.

$$P=\frac{1}{6}$$

B. Roll a d8 and a d12 and get two 1s.

$$P=\left(\frac{1}{8}\right)*\left(\frac{1}{12}\right)=\left(\frac{1}{96}\right)$$

C. Roll a d4 and a d20 and get two 4s.

$$P=\rule{4cm}{0.4pt}$$

D. Roll three d6 dice and get all 6s.

$$P=\rule{4cm}{0.4pt}$$
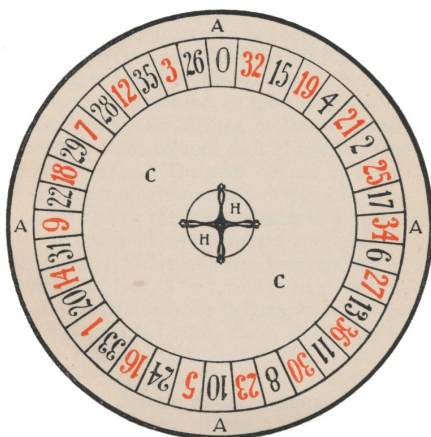
A roulette wheel. Picture by Toni Lozano.

Roulette

Roulette is a gambling game. There is a wheel and a ball. Players make a bet on a number or pattern. The ball is rolled, and it stops on a number. If a player's bet is good, the person wins some money. If not, they lose some.

Math Questions

Write your answers.

1. If you bet $1 on ① and you win, you get $36.

   What is the probability of a ①?   _____

2. If you bet $1 on ⑧ and you win, you get $36.

   What is the probability of an ⑧?   _____

3. If you bet $1 on RED, and you win, you get $1.

   What is the probability of an RED? _____

4. If you bet $1 on EVEN and you win, you get $1. (EVEN = 2, 4, 6, 8, …)

   What is the probability of an EVEN? _____

Strategy Questions

Think about the answers.

1. Do you want to play Roulette?

2. If you play Roulette, do you want to win slowly or quickly? Why?

3. If you play Roulette for a long time, what will probably happen?

Chapter 3: Arithmetic

Drills

Answer the questions.

1. $5+5=$ _____

2. $2\times16=$ _____

3. $64\div8=$ _____

4. $12+24=$ _____

5. $99-7=$ _____

6. $2\times2\times2=$ _____

7. $\sqrt{9}=$ _____

8. $10^{10}=$ _____

9. Seven plus ten equals _____.

10. Nine times six equals _____.

11. Four minus one equals _____.

12. Thirty divided by five equals _____.

13. One hundred minus one is _____.

14. Twenty over four is _____.

15. Five squared equals _____.

16. The square root of sixteen is _____.
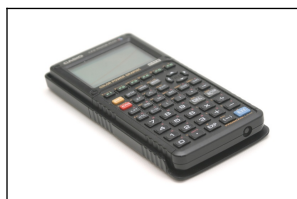
A graphing calculator.

Word Problems

Example   *Mary bought ten red apples, three green apples, and six oranges. How many pieces of fruit does she have?*

   Expression: 10+3+6   Answer: 19 pieces

1. *Tom has three black T-shirts. He has four more white T-shirts than black T-shirts. How many shirts does he own?*

   Expression: _____   Answer: _____

2. *Aaron made thirty-six cookies. He ate one and gave ten to Beth. How many cookies does he have now?*

   Expression: _____   Answer: _____

3. *Beatrice studied for 30 minutes. Then she watched TV for an hour. After that, she spent twenty minutes eating dinner. How long did that all take?*

   Expression: _____   Answer: _____

Word Problem Making

Make questions for the expressions.

Example   Expression: $50-(2\times10)-3$

Question: *Max had $50. He bought two $10 watermelons. Then he bought a bag of oranges for $3. How much money does he have now?*

1.   Expression: $12-3-2$

Question: _____

_____

2.   Expression: $0:30+1:00+0:15$

Question: _____

_____

Buy a Cupcake

A package of cupcakes costs $3.58. How can you pay for it?

| dollar | half dollar | quarter | dime | nickel | penny |
| $1.00 | $0.50 | $0.25 | $0.10 | $0.05 | $0.01 |

Answer	dollars	quarters	dimes	nickels	pennies	**Total**
Example	2	6	0	1	8	$3.58
1.						
2.						
3.						
4.						

Chapter 4: Equations

Numbers to Six

Consider the formula 2 2 2=6 . If you add symbols, you get the equation 2+2+2=6 . By writing symbols – but no numbers – can you make equations for the other numbers?

1 1 1=6	2 2 2=6	3 3 3=6	4 4 4=6	5 5 5=6
6 6 6=6	7 7 7=6	8 8 8=6	9 9 9=6	10 10 10=6

Word Problems

Write the equation and solve it.

Example *Edna's water costs $5 a month, plus $0.05 for each liter of water. Write an equation for **G**, her monthly bill. Suppose she uses 73 liters in January. What is her January bill?*

Equation:　　$G=\$5.00+(\$0.05\times w)$　　Solution: _____

1. *The cost to rent a car is $50.00 per day. Write an equation for **R**, rental costs. Meghan rented a car for five days. How much did she pay?*

Equation: _____　　Solution: _____

2. *Danielle has a cell phone. The monthly service costs $12.60. She pays an extra $0.25 per minute. Write an equation for **C**, her total monthly bill. In December, her total bill was $21.10. How many minutes did she talk?*

Equation: _____　　Solution: _____

3. *Travis likes to buy apples and peaches. Apples cost $0.75 each, and peaches cost $1.12 each. Write an equation for **F**, his fruit bill. If he buys four apples and twice as many peaches, how much does it cost?*

Equation: _____　　Solution: _____

Equation Comprehension

Practice listening to and saying the equations.

A.　$y=x$	D.　$y=\dfrac{1}{x}$	G.　$a=(\dfrac{1}{2})bh$
B.　$y=5x+2$	E.　$y=\dfrac{2}{3x}$	H.　$a=\pi r^2$
C.　$y=ax+b$	F.　$y=3x$	I.　$A=4\pi r^2$

Number Brain Teasers

Write the missing words.

1. 12 = M in a Y. M: _____ Y: _____

2. 26 = L of the A. L: _____ A: _____

3. 52 = C in a D. C: _____ D: _____

4. 7 = D in a W. D: _____ W: _____

5. 4 = S in a Y. S: _____ Y: _____

6. 47 = P in J. P: _____ J: _____

7. 525,600 = M in a Y. M: _____ Y: _____

8. 257 = C in the W. C: _____ W: _____

9. 50 = S in the U.S. S: _____ U.S.: _____

Make Your Own Brain Teaser

10. _____ _____ _____

11. _____ _____ _____

0 1 2 3 4 5 6 7 8 9 0 5

Break the Code

```
   23 5    1 18 5    23 8 1 20    23 5    20 8 9 14 11 .

  1 12 12    20 8 1 20    23 5    1 18 5    1 18 9 19 5 19

    23 9 20 8    15 21 18    20 8 15 21 7 8 20 19 .

    23 9 20 8    15 21 18    20 8 15 21 7 8 20 19 ,

     23 5    13 1 11 5    20 8 5    23 15 18 12 4 .
```

– Siddharta Gautama, 563-483 B.C.

Demonstrations I: Numbers

RULES

In pairs or individually, make and give a math demonstration.

Presentations should be two to three minutes long.

Explain the topic and show examples on the chalk board.

KEY POINTS

Clarity ▪ Volume ▪ Pronunciation ▪ Chalkboard ▪ Time

TOPICS

A) Describe a foreign country's money system.

B) What three neighboring numbers add up to 72?

C) Explain how to read and write Mayan numerals.

D) Explain how to read and write Babylonian numerals.

E) Talk about Greek letters that are used in mathematics.

F) Why is October the tenth month?

G) Why are there twenty-four hours in a day?

H) Teach the class how to count to ten in a language they don't know.

I) Two trains leave Budapest. One goes west at 50 km/h. The other goes east at 30 km/h. How far apart are they after two hours and ten minutes?

J) Lindsay watched TV on Saturday and Sunday for a total of 4 hours. On Sunday she watched for twice as long as Saturday. How long did she watch on Saturday?

K) Ken has an empty pool. It can hold 9000 buckets of water. He can carry 3 buckets a minute. Barbie can carry 2 buckets a minute. How long will it take to fill the pool?

L) Jeff is 12 kg heavier than Kim. Their total mass is 143 kg. How much does Jeff weigh?

Part II: Statistics

One Week

Weekly Consumption
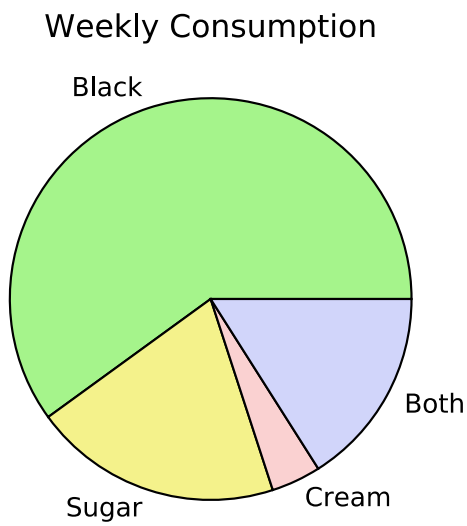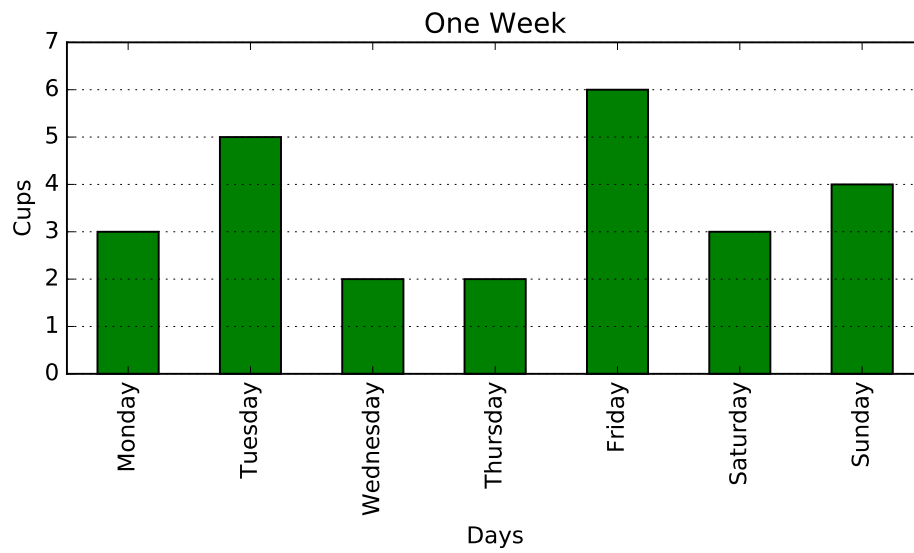
Chapter 5: Averages

Graph Comprehension

Listen and identify the charts.

A.	B.	C.	D.
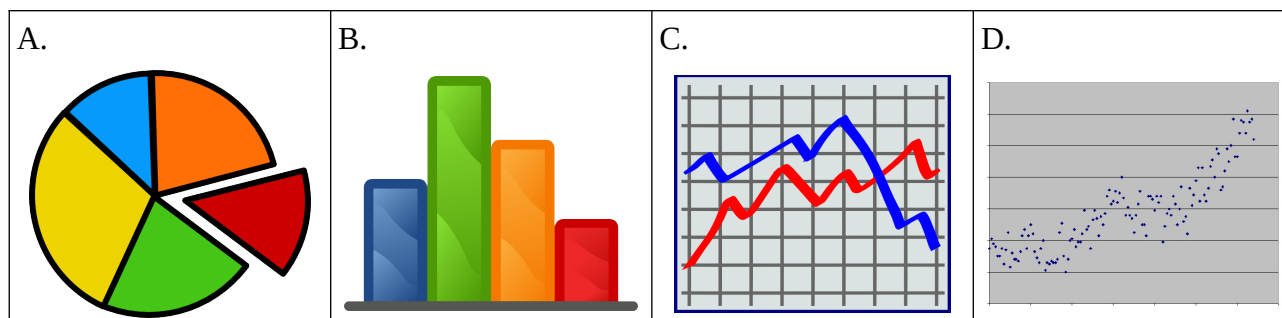
Three Kinds of Averages

Mean. The total divided by the count. $[1 , 3 , 5]$ $\text{mean} = \dfrac{1+3+5}{3} = 3$

Median. The middle number. $[145 , 148 , 149 , 150 , 200]$ $\text{median} = 149$

Mode. The most common number. $[3 , 5 , 5 , 7 , 9 , 15]$ $\text{mode} = 5$

Calculate the Averages

1. $[4 , 8 , 9 , 12 , 20 , 20 , 21 , 27, 28]$

Mean: _____

Median: _____

Mode: _____

2. $[28 , 28 , 28 , 32 , 55 , 56 , 57 , 59 , 62]$

Mean: _____

Median: _____

Mode: _____

Minimal Pair Listening

Circle the word you hear.

1.	pie / buy	6.	value / very
2.	base / vase	7.	thing / sing
3.	chart / shirt	8.	image / imagine
4.	slice / slices	9.	product / produce
5.	cone / comb	10.	median / medium

Interview Your Classmates

What size are your classmates' shoe sizes? Ask them and take notes here.

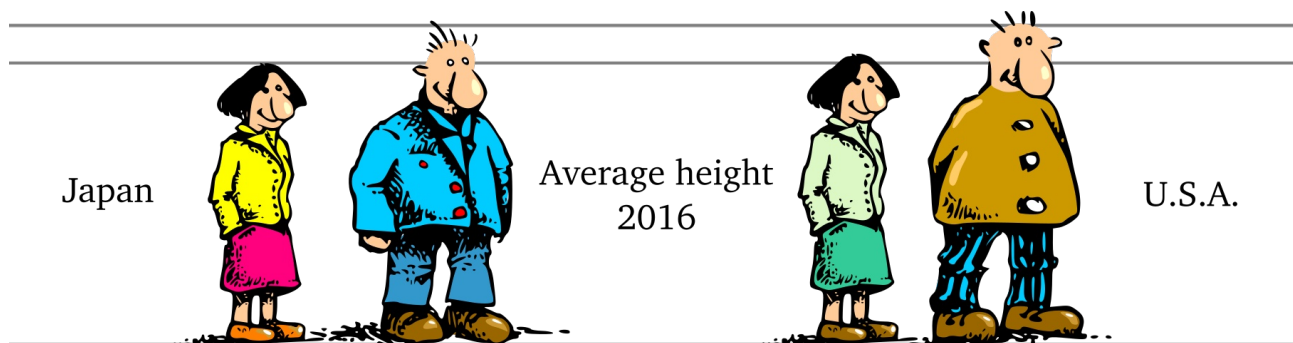## Calculate the Class Average Shoe Size

Mean: _____

Median: _____

Mode: _____

Japan   Average height 2016   U.S.A.

Average Height in History

Many people believe that the average height of humans is going up over the years, but actually it has varied over the centuries. Average height is affected by climate, cities, war, and population. One thousand years ago, the average height for men was 172.7 cm. In the 1600s and 1700s, it went down to 167 cm. In 2016 in America, the average for men was 178.2 cm and women was 164.1 cm. In the same year in Japan, the average for men was 171.2 cm and women was 158.8 cm.

1. *On average, men are taller than women.*   TRUE / FALSE

2. *People were taller 1000 years ago than 400 years ago.*   TRUE / FALSE

3. *American women are taller than American men.*   TRUE / FALSE

4. *Human height has changed a lot in history.*   TRUE / FALSE

5. *Human height has always gone up.*   TRUE / FALSE

Chapter 6: Birthday Frequency

Birthday Months

Ask your classmates when their birthday is, and write down the numbers for each month.

Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec

1. Which month has the most birthdays in your class? _____

2. Some months have 31 days, some months have 30 days, and one has 28 days. For your class, do the longest months contain the most birthdays? _____

3. The below table shows birthday by month from the U.S.A. over an eight year span. What month had the most birthdays? _____

4. What month had the fewest birthdays? _____

U.S.A. Births by Month 1995-2002
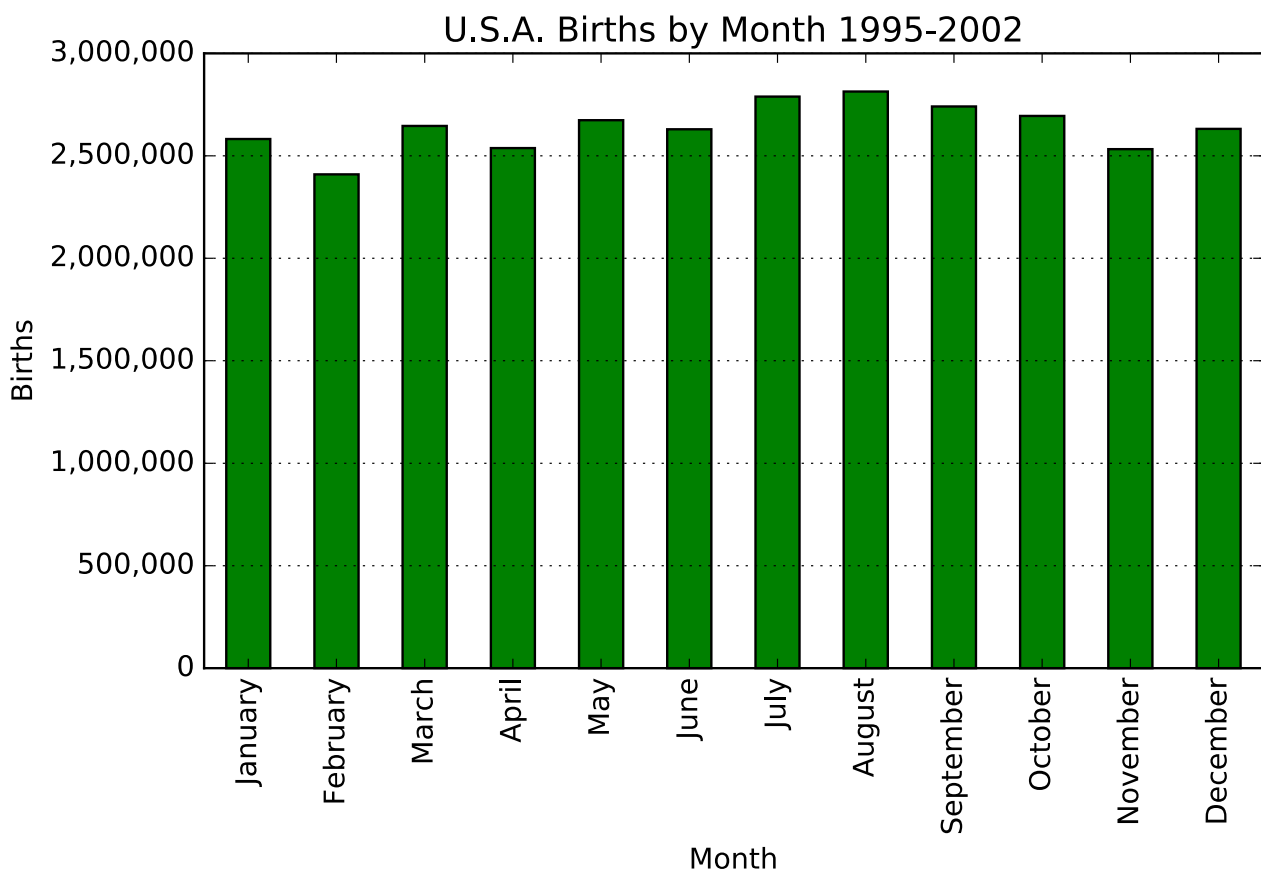
Data from the National Center for Health Statistics.

Birth Times

According to [2015] data, American mothers-to-be aren't having too many late night hospital visits. A study by the U.S.A.'s National Center for Health Statistics found that the highest percentage of births took place between the hours of 8:00 a.m. and noon. Less than 3% of babies were born each hour from midnight to 6:59 a.m. There is a peak of deliveries in the morning around 8 o'clock– before doctors attend to other patients–and then again at noon, before physicians see their afternoon patients. If births were equally timed throughout the day, an average of 4.2 percent of newborns would be delivered each hour. But based on the latest data, the highest percentages of births occurred during the 8 a.m. (6.3%) and noon (6%) hours.

– Adapted from [What time of day are most U.S. babies born?](#) By Lena H. Sun, May 8, 2015.

Middle Day

Imagine Jack was born on May 12th and Jill was born on August 8th. Forget about the month, and remember the number. The average of 12 and 8 is 10. Let's consider more people.

- What is the middle day of this month?

- What is the average middle day of the months in a year?

- Do you think the average birthday is on the average day of the month? Why?

Astrology

In Western countries, some people feel that there is a connection with the month and day of your birth and your personality and luck. Look for your birthday to find your astrological sign. Does it match your personality?

	Capricorn	12/22 1/20	Stubborn, patient, hard-working.	**Cancer**	6/22 7/23	Compassionate but moody.
	Aquarius	1/21 2/19	Original, creator, trend-setter.	**Leo**	7/24 8/23	Lucky, optimistic, charismatic.
	Pisces	2/20 3/20	Mysterious, confusing.	**Virgo**	8/24 9/22	Fussy but strong and creative.
	Aries	3/21 4/19	Adventurous, naive active, outgoing.	**Libra**	9/23 10/22	Desires harmony and balance.
	Taurus	4/20 5/20	Cool, calm and collected exterior.	**Scorpio**	10/23 11/22	Wise, powerful.
	Gemini	5/21 6/21	Always curious and mischievous.	**Sagittarius**	11/23 12/20	Optimistic and adventurous.

Chapter 7: World Statistics

2014 Foreign Resident Population of Japan
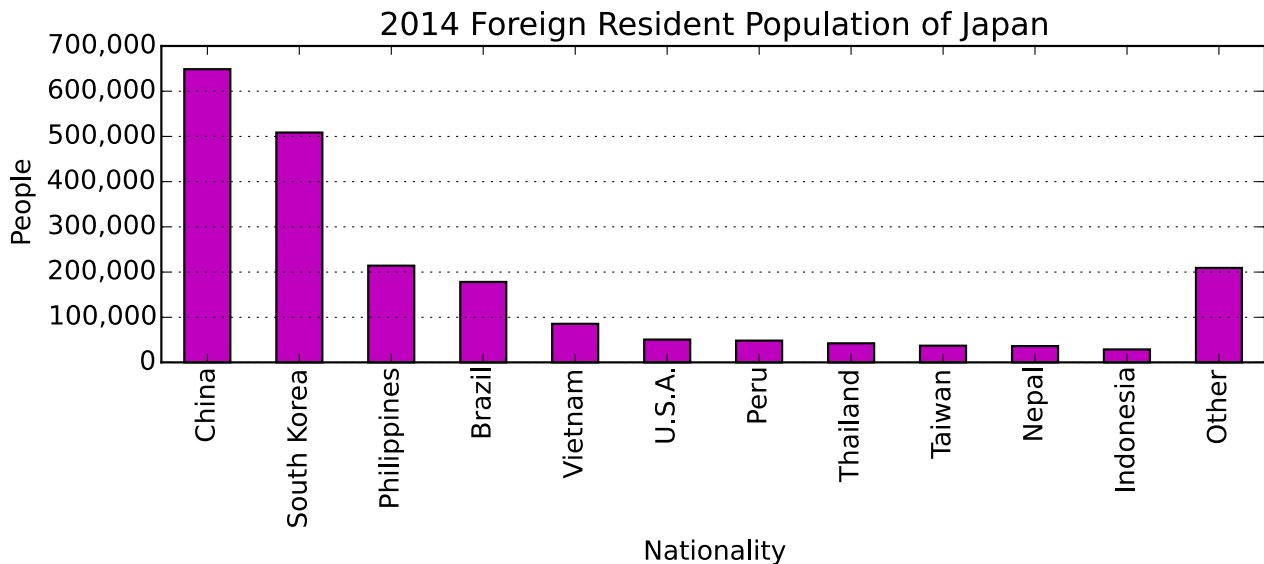
True/False Questions

1. There are more people from China than any other country.   TRUE / FALSE

2. Brazilians are fewer in number than Americans.   TRUE / FALSE

3. There are around 200,000 Filipinos.   TRUE / FALSE

4. There are less than 50,000 Nepalese.   TRUE / FALSE

5. There are about as many Americans as Peruvians.   TRUE / FALSE

Ice Cream Questions

Favorite U.S. Ice Cream Flavors (2013)
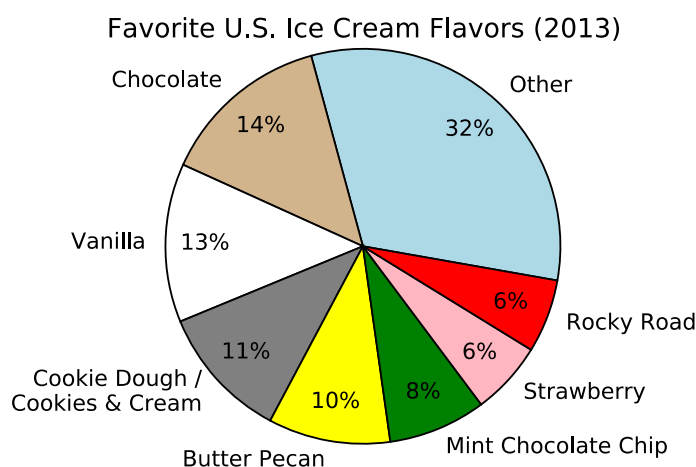
1. What is the most popular flavor?

   _____

2. What is the least popular flavor?

   _____

3. Are you surprised by these?

   _____

4. What's your favorite flavor?

   _____

U.S. Historical Population 1790-2015

Graph Reading

1. What was the population in the year 1880? _____

2. When was the population 250 million? _____

3. When was the highest population? _____
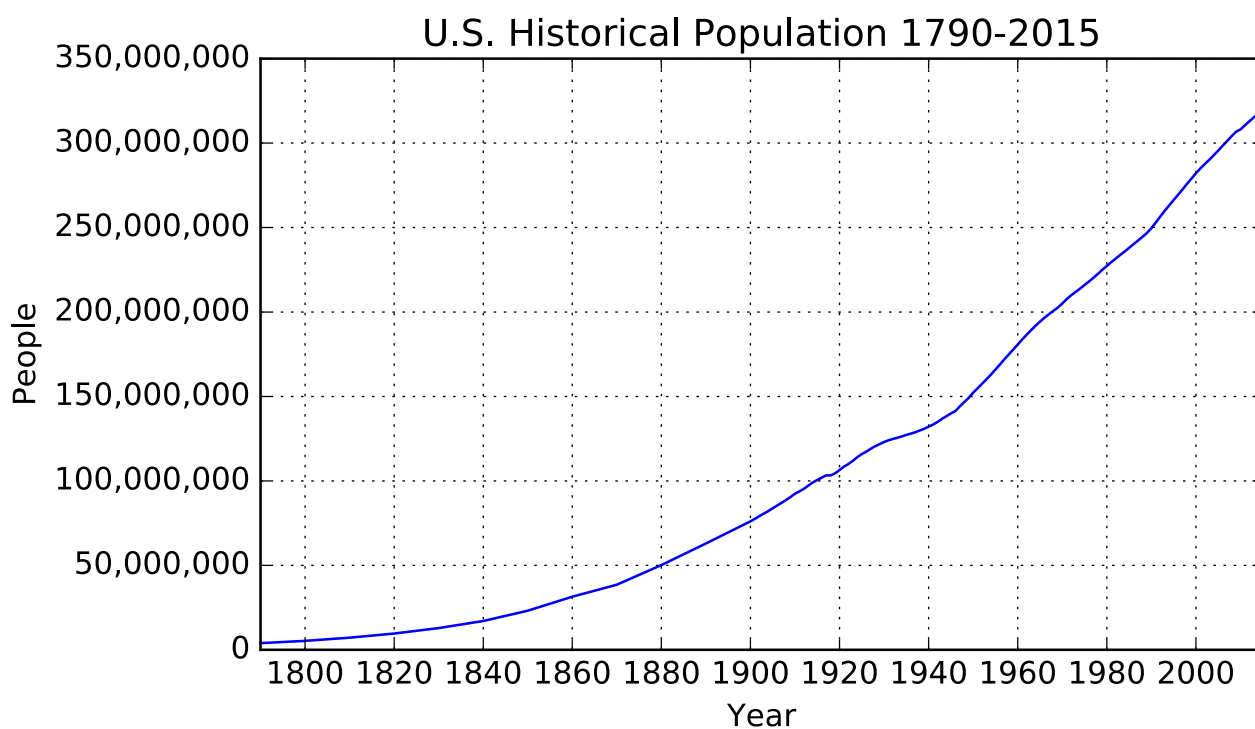
4. What happened around 1930–1940? _____

Restate the Sentences

Rewrite the sentences keeping the general meaning the same.

Example *70% of the Earth's surface is water.* → *30% of the Earth's surface is land.*

Example *Mount Everest is taller than K2.* → *K2 is shorter than Mount Everest.*

1. *The world population is 49.3% women.* → _____

2. *Japan's population is 51.3% women.* → _____

3. *Africa is larger than Europe.* → _____

4. *Japan is twice the size of Cambodia.* → _____

5. *Norway is colder than Thailand.* → _____

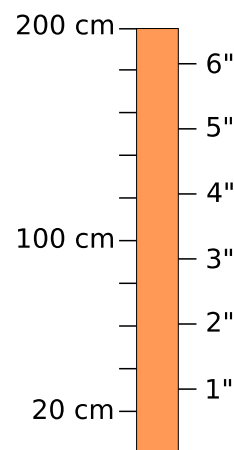6. *February isn't as long as other months.* → _____

Chapter 8: Measurement

Convert the Height

1 foot (1') = 12 inches (12")

1 foot (1') = 30.48 cm   1 inch (1") = 2.54 cm

1. Napoleon was 5' 7" tall. What's that in centimeters?   _____

2. Mayu Watanabe is 5' 1.42" tall. What's that in centimeters? _____

3. Kim Jong-un is 5' 8" tall. What's that in centimeters?   _____

4. LeBron James is 6' 8". What's that in centimeters?   _____

Challenge Questions

5. Saori Kimura is 185 cm tall. What's that in feet and inches? _____

6. Hello Kitty stands five apples tall. A standard-size apple in England is 3.8 inches tall. How tall is she in centimeters?   _____
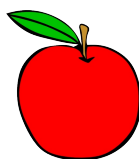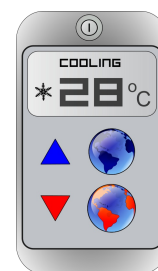
Temperature

In the U.S., people talk about temperature in degrees Fahrenheit (°F). Around the world, many people use degrees Celcius (°C). You can convert using this formula.

$$f = \frac{9}{5}c + 32$$

1. What is 0°C in °F?   _____

2. What is 0°F in °C?   _____

3. What is 100°C in °F?   _____

4. When is the temperature the same in both °F and °C?

_____

True/False Quiz

1. Water freezes at 0°C.   TRUE / FALSE

2. Water boils at 100°C.   TRUE / FALSE

3. A football field is around 6,000 square meters.   TRUE / FALSE

4. The Eiffel Tower is 524 meters high.   TRUE / FALSE

5. It is 845 miles (1,360 kilometers) from New York to Los Angeles.   TRUE / FALSE

6. The volume of an average American bathtub is around 70 liters.   TRUE / FALSE

7. There are around 525,600 minutes in a year.   TRUE / FALSE

Percent correct:_____

Number Prefixes

Many words start like this…

1	2	3	4	5	6	7	8
uni	*bi*	*tri*	*quad*	*quin*	*sexa*	*septem*	*octo*
mono	*di*	*tri*	*tetra*	*penta*	*hexa*	*hepta*	*octa*
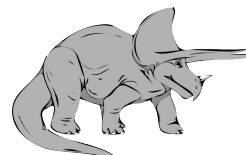
Brainstorming

Think of some words that begin with these prefixes.

uni-	bi-	tri-	oct-

Demonstrations II: Statistics

In pairs or individually, make and give a math demonstration.

Presentations should be two to three minutes long.

Explain the topic and show examples on the chalk board.

KEY POINTS

Clarity ▪ Volume ▪ Pronunciation ▪ Chalkboard ▪ Time

TOPICS

A) What are your classmates' favorite colors?

B) How much do the lunches in the cafeteria cost?

C) How many students are in each class at this school?

D) How many teachers teach each subject at this school?

E) How many classes of each subject do you have in a week?

F) How long does it take your classmates to come to school?

G) How many hours of sleep do your classmates get each night?

H) How many rooms on there on each floor of the school building?

I)   How many staircases are there, and how many steps do they have?

J)   How much time do your classmates spend each day watching TV?

K) How much time do your classmates use their phones each day?

L) How many brothers and sisters do your classmates have?

M) How many cell phones have your classmates owned in their lives?

26
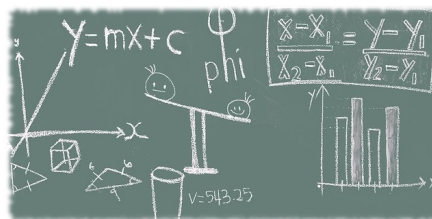
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